

Due: Monday August 19th by 11:45pm

All work and answers should be clearly written on this page. Please box or circle your final answer or place your answers in the given answer blanks below. The use of a calculator is NOT ALLOWED unless explicitly stated in the problems below. Email [karwood@wcpss.net](mailto:karwood@wcpss.net) AND [wbarnwell@wcpss.net](mailto:wbarnwell@wcpss.net) for directions on how to submit your assignment.

## Part 1: Algebra, Part 1

1. Solve for c:  $\frac{k}{c} = \frac{9}{c-p}$

2. Simplify each expression.

(a)  $10\frac{3}{7} - 6\frac{8}{13}$

(c)  $(3w - 4)(w^2 + 7w - 9)$

(b)  $4\frac{2}{5} \div 3\frac{3}{7}$

(d)  $\frac{5}{x+4} - \frac{7}{x-5}$

3. Simplify each expression by rationalizing the denominator

(a)  $\frac{8}{2+6i}$

(b)  $\frac{3}{7+\sqrt{5}}$

4. Using factoring, solve for m:  $30m^2 + 13m + 20 = 30$

5. Find the slope and the location of the y-intercept for the linear equation:  $3x + 5y = 18$

Slope:
Y-intercept:

6. Write the equation of a line perpendicular to  $3x + 5y = 18$  and through the point  $(15, -4)$ , in standard form. **(Please be aware that this is the same linear equation used in the previous problem)**

7. Find the domain and range of each of the following functions. Clearly label and give your answers in interval notation.

(a)  $p(x) = x^2 - 8x + 12$

Domain:
Range:

(c)  $f(x) = \frac{-7}{3}x + 8$

Domain:
Range:

(b)  $g(x) = 2\sqrt{x+6} - 5$

Domain:
Range:

(d)  $w(x) = 4 \cos\left(x - \frac{2\pi}{3}\right) + 2$

Domain:
Range:

8. Find the standard equation of the ellipse and identify its center, along with the locations of vertices and covertices.  $18x^2 + 12y^2 + 288x - 120y + 1236 = 0$ .

Center:
Vertices:
Covertices:

9. Solve the equation  $|4d - 53| = 77$ .

10. Write  $\frac{\frac{1}{y-k} - \frac{1}{y}}{k}$  as a simplified fraction.

11. Given the functions:  $f(x) = x^2 - 3x + 4$  and  $g(x) = 2x - 4$ .

(a) Evaluate  $f(g(x))$

(b) Evaluate  $g(f(x))$

12. **(Scientific Calculator Active - you must show all work!!)** Anna has been making runs to the nearby Starbucks for Frappacinos for her friends during smart lunch. On Monday she purchased 6 regular drink and 7 grande drinks for a total of \$61.32. On Tuesday, she purchased 10 regular drinks and 3 grande drinks for a total of \$56.96. What is the price for each regular drink and each grande drink?

13. Rewrite the function  $f(x) = 2|x - 3| + 6$  as piecewise function.

$f(x) = \left\{ \right.$
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14. Write the equation of a line parallel to  $6x - 11y = 32$  and goes through the point  $(-3, 8)$  in point-slope form.

15. Find the points of intersection of the given functions.  $f(x) = -4x^2 + 5x + 15$  and  $g(x) = -3x - 45$

16. Solve for p in simplest radical form:  $6p^2 - 48p + 15 = 0$

17. Solve for a and b:  $(a + bi)(7 + 4i) = -71 + 43i$

18. **(Scientific Calculator Active - you must show all work!!)** A rocket is launched vertically with an initial velocity of 392 feet per second. The formula  $h(t) = -16t^2 + 392t - 70$  gives the height of the rocket after t seconds. What is the maximum height of the rocket?

19. If a and b are both nonzero and  $\frac{a}{x-6} + \frac{b}{x+8} = \frac{7x+84}{(x-6)(x+8)}$  find a and b.

20. What is your favorite movie? Discuss why?

Part 2:

21. Solve for x:  $6^{x+8} - 103 = 61$

22. Solve for x:  $\log_6 2x + \log_6(x - 4) = \log_6 24$

23. Expand:  $\ln \left[ \frac{(7x^2 - 3)\sqrt{x+8}}{(3x-5)^6} \right]$

24. Condense:

$$5\log_3(2b + 1) + 9\log_3 w - (2\log_3(b - 7) - \log_3 w)$$

25. Solve for x:  $\frac{e^{8x-5}}{e^{2x-9}} = 68$

26. Find the domain and range of each of the following functions. Clearly label and give your answers in interval notation.

(a)  $g(x) = \ln(x - 7) + 3$

Domain:

Range:

(b)  $h(x) = 3e^{x+5} - 6$

Domain:

Range:

27. Find the equation of the line that goes through the point  $(8, -7)$  and the x-intercept of the function  $y = \ln(x - 5)$ .

28. Solve for k:  $(3^{5k})^3 = (3^4)^{(k+7)}$

30. Solve for d:  $(e^2)^{5d} = (e^6)(e^{4d})$

29. Solve for a:  $6^{2a} + 2 \cdot 6^a - 63 = 0$

31. Solve for x:  $e^{(2\ln x - \ln(x^2 - 4x - 7))} = 3$

32. **(Calculator Active – you must show all work!!)** An adult takes 800 mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about 42%. How much ibuprofen is left after 9 hours?

33. Find the inverse of the function:  $f(x) = 4^{x+6} - 13$

34. **(Scientific Calculator Active – you must show all work!!)** From 2000 to 2008 the value of the dollar has been shrinking. The value can be modeled by  $V = 1.37(0.948)^t$ , where  $V$  is the value of the dollar and  $t$  is the number of years since 2000. In what year is the dollar worth \$1.03?

### Part 3: Trigonometry

35. **(Calculator Active – you must show all work!!)** A patrol boat finds the angle of elevation of a nearby volcanic island peak is  $17.9^\circ$ . After the boat moves 500 m closer, the angle of elevation is  $25.4^\circ$ . Find the height of the peak above sea level.

36. **(Calculator Active – you must show all work!!)** The town of Fishkill is connected to the towns of Beacon and Poughkeepsie by straight highways. The angle between these highways measures  $63^\circ$ . How far apart are Beacon and Poughkeepsie if their distances from Fishkill are 12 miles and 18 miles, respectively.

37. **(Calculator Active – you must show all work!!)** Coach Morris sits in a tower directly over a practice field that is 90 yards long. The angles of elevation from the end lines of the practice field are  $35^\circ$  and  $65^\circ$ . How far is Coach Morris from the nearer end of the practice field?



38. **(Calculator Active – you must show all work!!)** In  $\triangle ABC$ ,  $b = 19$  mm, and  $c = 22$  mm, and  $m \angle A = 52.7^\circ$ . Find the area of the triangle.

39. The terminal side of an angle in standard position passes through the point  $(5\sqrt{3}, -5)$ . Find the measure of the angle for  $0 \leq x < 2\pi$ .

40. Find the exact values (in radians) of each of the following angles.

a.  $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) =$

b.  $\sin^{-1}(-1) =$

c.  $\cot^{-1}(-1) =$

d.  $\sin^{-1}\left(\frac{-1}{2}\right) =$

e.  $\tan^{-1}(0) =$

41. Write the given expression in algebraic form:  $\tan\left(\arccos\left(\frac{x+2}{5}\right)\right)$

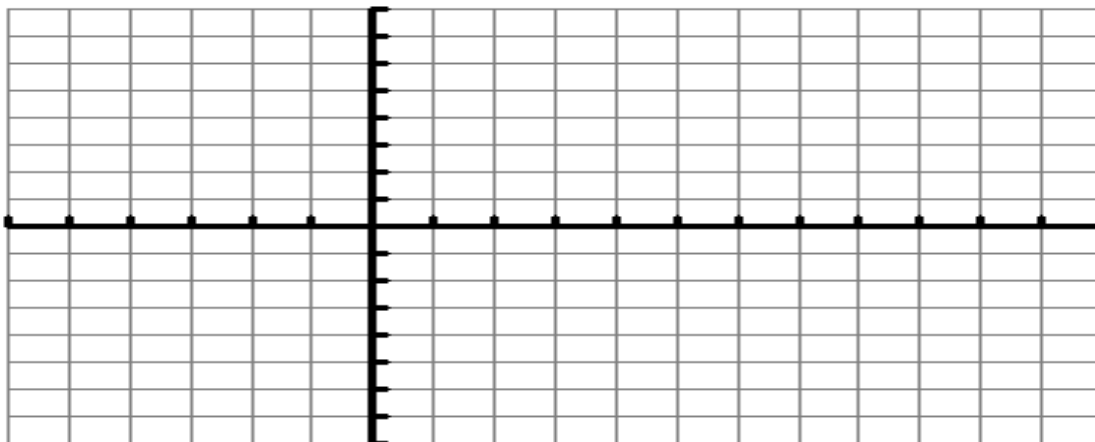
42. Write the polar form of the equation  $x^2 + 4y^2 - 3x = 0$ . (Note: you do not need to solve for  $r$ .)

43. Solve the equation for  $0 \leq x \leq 2\pi$ .  $2 \sin(2x) \cos x = 3 \sin x$

44. Give the general solution for the equation in radians:  $\sec^2 x + \tan^2 x = \sec x$

45. Identify the amplitude, period, phase shift, and vertical shift, then use them to graph the at least 2 full periods of:  $y = -3 \cos \pi(x + 1) + 4$  (Please include necessary labels for the graph and axes below).

Amplitude: \_\_\_\_\_ Period: \_\_\_\_\_ Phase Shift: \_\_\_\_\_ Vertical Shift: \_\_\_\_\_



46. Prove:  $\frac{\csc x - 1}{\cot x} - \frac{\cot x}{1 + \csc x} = \frac{2 \cos x}{1 + \sin x}$

47. Prove:  $1 - \tan^4 \theta = \frac{\cos 2\theta}{\cos^4 \theta}$

#### Part 4: Limits and Continuity

48. Find each limit. If the limit does not exist, state DNE.

a.  $\lim_{x \rightarrow 3} 2x^2 - 5x + 6$

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b.  $\lim_{x \rightarrow 2} \frac{4x + 8}{x^2 - 4}$

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c.  $\lim_{x \rightarrow 5} f(x)$ ,  $f(x) = \begin{cases} 2x + 7 & x < 5 \\ x^2 - 8 & x \geq 5 \end{cases}$

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d.  $\lim_{x \rightarrow -5} \frac{-5}{x^2 + 10x + 25}$

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e.  $\lim_{x \rightarrow 4^-} \frac{x - 4}{|x - 4|}$

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f.  $\lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x}$

\_\_\_\_\_

g.  $\lim_{x \rightarrow -3} \frac{2x + 6}{x^3 + 27}$

\_\_\_\_\_

h.  $\lim_{x \rightarrow 2\pi^-} \csc x$

\_\_\_\_\_

i.  $\lim_{x \rightarrow 0} \frac{x}{2 - \sqrt{4 - x}}$

\_\_\_\_\_

j.  $\lim_{x \rightarrow \infty} \left(\frac{1}{3}\right)^x$

\_\_\_\_\_

k.  $\lim_{x \rightarrow \infty} \frac{x + 5}{3x^4 - 6x}$

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l.  $\lim_{x \rightarrow \infty} 3^x$

\_\_\_\_\_

m.  $\lim_{x \rightarrow \infty} \frac{3x^2 + 7}{5 - 6x^2}$

\_\_\_\_\_

n.  $\lim_{x \rightarrow \infty} \log \left[ \sec \left( \frac{1}{x} \right) \right]$

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49. Determine the points of discontinuity (if any) for each function. For each point of discontinuity, state whether the discontinuity is removable or non-removable.

a.  $f(x) = \frac{x-5}{x^2-7x+10}$

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b.  $f(x) = \llbracket x - 3 \rrbracket$

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c.  $f(x) = \frac{x+3}{x^2+3}$

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50. Find all vertical, horizontal, and slant asymptotes for each rational function.

a.  $f(x) = \frac{4x-16}{x^2-x-12}$

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b.  $f(x) = \frac{x^3+27}{x^2+4x-12}$

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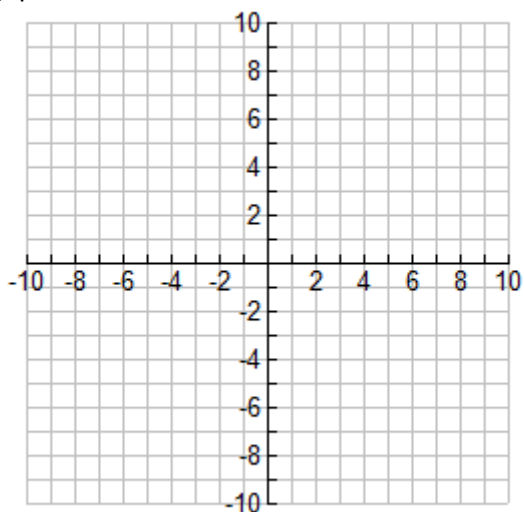
c.  $f(x) = \frac{x+5}{18-9x}$

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51. Graph the piecewise function below and answer the corresponding questions.

$$f(x) = \begin{cases} -7 & x < -7 \\ -(x+5)^2 + 1 & -7 \leq x \leq -2 \\ 2x - 4 & -2 < x < 4 \\ 6 & x = 4 \\ |x - 6| + 2 & 4 < x \end{cases}$$



**Discuss** the continuity of the function at each x value using the **3 part definition of continuity ONLY!**

a)  $x = -7$

b)  $x = -2$

c)  $x = 4$

52. Find the values of a and b that allow the function to be continuous.

$$f(x) = \begin{cases} 7x - 2, & x \leq 1 \\ ax + b, & 1 < x < 4 \\ 3x + 5, & x \geq 4 \end{cases}$$

53. What has been your favorite summer activity thus far? Discuss why.

**Part 5: Algebra #2**

54. A dog owner wants to make a rectangular pen for her dogs to play in. This enclosure will use a wall from the side of their home as one side and 3000 meters of fencing for the other three walls. Find the dimensions of the dog pen that give the greatest area and state the area.

Dimensions:

Area:

55. Solve the equation:  $\frac{5}{x-8} - \frac{6}{x-3} = \frac{10}{x^2-11x+24}$

56. Use long division to divide the polynomials and write your result in fractional form:

$$(x^4 - 3x^3 + 3x^2 - 6x + 4) \div (x^2 + 2)$$



57. Use synthetic division to divide the polynomials. Write the result in fractional form:

$$(x^4 - 6x^3 + 3x - 12) \div (x - 5)$$

58. List all the possible rational zeroes given by the Rational Root Test for the given polynomial:

$$f(x) = 12x^3 + 5x^2 + 18x + 8$$

59. Evaluate  $(5x - 3)^4$

60. Given the polynomial  $f(x) = -(x + 5)^3(x - 6)(x - 3)^2$  for each question below give an answer and justification.

(a) Where are the x-intercepts?

(b) For each x-intercept you listed in part (a) describe the behavior of the graph at that point (ex. Goes through the point, changes direction, etc.)

(c) What is the right end behavior?

(d) What is the left end behavior?

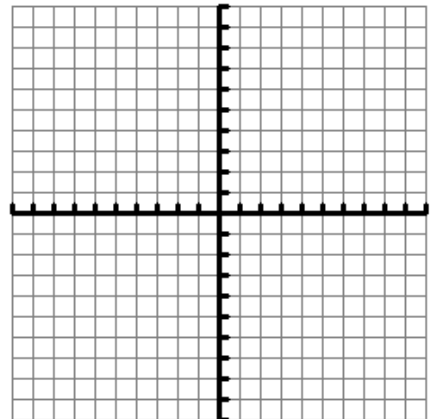
61. The polynomial  $f(x) = x^3 - 36x$  has three x-intercepts  $(-6,0)$ ,  $(0,0)$  and  $(6,0)$ . Identify and justify what type of extrema (relative minimum, relative maximum or neither) is located between the given intervals:

(a)  $-6 < x < 0$

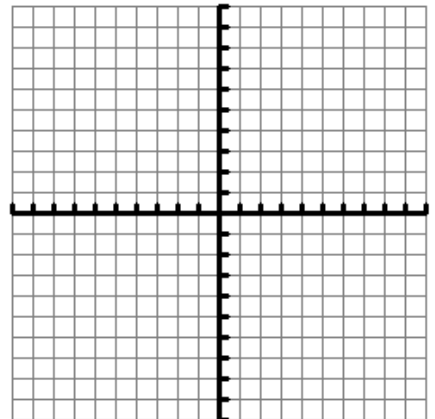
(b)  $0 < x < 6$

(c)  $6 < x < 10$

62. Graph the functions  $f(x) = -2(x - 3)^2 + 5$ ,  $g(x) = 3|x - 2| - 6$  on the coordinate axes given below and shade in the enclosed region(s) generated by these graphs. Show all necessary work, if any, you used to create the graphs below.



63. Graph the following system of inequalities  $6x - 5y \leq -8$ ,  $y \leq 4$ , and  $x \geq -8$  on the coordinate axes given below and evaluate the area of this feasible region. Show all necessary work, if any, you used to create the graphs below.



64. If  $f(x) = 3x^2 + 8x - 6$  evaluate the limit below.

$$\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

65. Series Review Question

a) Give the series in expanded form and evaluate:  $\sum_{k=1}^7 (k^2 + 5k - 8)$

b) Give the series in expanded form and evaluate:  $\sum_{k=1}^7 k^2 + 5\sum_{k=1}^7 k - \sum_{k=1}^7 8$

c) Recall the following finite sums formulas:

(sum of constants) $\sum_{k=1}^n c = cn$	(sum of squares) $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$
(sum of integers) $\sum_{k=1}^n k = \frac{n(n+1)}{2}$	(sum of cubes) $\sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^2(n+1)^2}{4}$

i) Using the finite sums formulas evaluate:  $\sum_{k=1}^7 (k^2 + 5k - 8)$

ii) Using the finite sums formulas evaluate:  $\sum_{k=1}^{14} (k^3 - 4k^2 + 5k - 14)$